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A Worthwhile Mathematical Task for Students and Their Teachers

WORTHWHILE MATHEMATICAL TASKS not only prompt students to learn mathematics, they also prompt teachers to learn and improve their teaching in their own mathematics classrooms. When teachers use worthwhile tasks, they have to learn "what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge" (NCTM 2000, p. 19).

This article describes a professional development experience in which teachers learned this way of teaching while implementing a worthwhile mathematical task for their students *and* for themselves.

Six teachers from a local middle school took part in a professional development exercise designed to help them implement worthwhile mathematical tasks known as Kid Case Studies, which also involved discussing their students' mathematical thinking as revealed in written work. The teachers attended after-school workshops (where they worked collaboratively on the students' tasks, discussed anticipated student responses, and addressed implementation issues), implemented the tasks with students in their own classrooms, and engaged in collaborative follow-up activities based on analyzing students' written responses to the tasks.

During the professional development experience, it became clear that the teachers were learning some valuable ideas about teaching. First, this article will introduce one of the Kid Case Studies the teachers used, titled "Departing On-Time." Then, the teachers' professional development experience will be described, using students' written work on the task, to illustrate how the teachers broadened their ideas about teaching.

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A Worthwhile Mathematical Task for Students

KID CASE STUDIES ARE THOUGHT-REVEALING activities for middle school students that are modeled after case studies frequently used in graduate programs and professional schools (Lesh, Hoover, Hole, Kelly, and Post 2000). They are designed to help students develop conceptual foundations for deeper and higher-order ideas in precollege mathematics. The Kid Case Study used in this article, titled "Departing On-Time," and other Kid Case Studies were developed with support from the School Mathematics and Science Center at Purdue University under the direction of Richard Lesh. Copyright is held by the School Mathematics and Science Center, but permission to photocopy is granted for classroom use and research.

It is recommended that you work through the "Departing On-Time Kid Case Study" in **figure 1** before continuing to read. To implement the activity, most teachers had their students work in groups of three for one forty-five-minute period. During this time, the students had access to computers. Thus, they typically performed their calculations by hand on paper and then used the word processing capabilities of the computers to type their letters. The teachers then devoted another class period to group presentations and follow-up class discussions about the different solutions and the mathematics embedded in the problem.

The "Departing On-Time Case Study" provides students with conceptual foundations for statistical topics such as average, outlier, spread, frequency, and standard deviation. All Kid Case Studies are designed to require students to interpret real-world situations and to formulate mathematical methods for making decisions for realistic clients. Here, students create a generalized procedure for the Spanish Club for ranking airlines in terms of most to least likely to depart on time. One characteristic of Kid Case Studies is that students' responses to the problem statement reveal their approach to the problem, providing excellent opportunities for teachers to analyze and reflect on their students' thinking.

A Worthwhile Task for Teachers

MOST OF THE TEACHERS IN THE CASE STUDY project were faced with new teaching experiences while implementing and analyzing students' work from the Kid Case Studies. The act of teaching using these case studies was different from most teachers' experiences, and this type of teaching was also unfamiliar, and sometimes frustrating, for the students. It was tempting to just teach how to solve the problem and thus decrease the problem-solving demand

for the students. Interpreting students' work was another new experience. Recognizing and understanding multiple ways of appropriately responding to the problem statement were unfamiliar. Further, responses were not simply right or wrong but fit along a continuum from unacceptable to effective. The teachers' professional development experience, particularly with "Departing On-Time," provides an

Problem Statement: In June, Ridgewood High School's Spanish Club is going on a study abroad trip to Venezuela. When traveling to Barcelona last year, their connecting flight to Reykjavik, Iceland, was late. They missed their connecting flight and had to stay overnight in the airport! This year the club is being more careful. They have identified five airlines that fly from O'Hare Airport to Venezuela, but they are still identifying more airlines. The flights all have a connecting flight in Mexico City. The students want the airline with the smallest chance of departing late from O'Hare so they can catch their connecting flight. In the table below is information about departure times (in minutes) for flights leaving O'Hare and arriving in Mexico City. Develop a procedure for ranking the five airlines from most to least likely to depart on-time. Describe your procedure in a letter to the Spanish Club so they may use your procedure with additional airlines.

Sky Voyage Airline	Central American Airlines	Mexico Express	Sudamerica Internacional	Southeast Airline
5	15	9	0	0
0	9	5	25	5
20	4	5	0	0
5	0	5	9	9
0	0	125	0	40
6	14	10	0	0
0	20	5	4	5
0	15	10	0	25
15	16	0	35	10
0	0	4	0	30
0	0	10	0	12
7	15	10	10	0
0	10	10	5	0
5	10	9	55	10
40	25	7	0	9
4	5	12	0	5
0	20	5	0	0
0	15	0	17	27
0	11	10	5	11
0	12	7	0	0
3	0	13	65	30
60	5	0	5	5
5	0	0	0	0
0	30	10	0	4
7	4	5	2	40
0	5	4	0	0
0	10	6	0	15
123	10	5	75	0
0	25	7	0	6
5	4	5	0	9

Fig. 1 "The Departing On-Time Case Study"

Dear Spanish Club,

We have been looking over various Airlines' flight times from the month of June in 1999. We added up the total amount of minutes they were late and divided it by 30 to find the average amount of minutes a particular airline was late per day. We would recommend that your first airline choice should be Southeast Airlines. Their total amount of minutes late in June was 307, and their average was 10.2 minutes. If you can't get tickets on that airline, then our second suggestion would be Sky Voyage. They only had a total of 310 minutes late in June for an average of 10.3 minutes late per day. We would advise you to fly Sudamerica for your third choice. They had 312 minutes late in June, and with an average of 10.4 minutes per day. Our second to last suggestion for you would be Central American. They only have a total of 314, which is only two more minutes than Sudamerica, but none the less it has more. Its average is 10.46 minutes late per day. At last we come to Mexico Express. They had 328 minutes late in June giving them an average of 10.9 minutes per day.

These are our suggestions and we hope they help you.

Fig. 2 Finding the average number of minutes late

opportunity to explore what the teachers learned about their students' thinking and about teaching.

Teachers' learning about students' thinking

The teachers noticed that when working on Kid Case Studies, the students proceeded through cycles of posing a solution, noticing aspects in need of improvement, and then refining their approach. During the "Departing On-Time Case Study," many of the students began by finding the average number of minutes late per flight for each airline (see the student work excerpt in **fig. 2**). (Note: For Central American Airlines and Mexico Express, the total number of minutes late is 309 and 313, respectively. Although the student has minor calculation errors, the reasoning is accurate.) However, many students revised their solution, because they recognized that the average number of minutes late for each airline only differed by approximately one-tenth of a minute or because one or two very late departures by one airline was equal to a large number of minimally late departures by another airline. Most of these students then revised their approach to looking at the number of on-time flights for each airline, as seen in **figure 3**.

Dear Spanish Club,

We have done some research to help you pick which airline to fly. The best airline to fly is Sudamerica Internacional. Here are the rest from best to worst: Sky Voyage, Southeast, Central America, and Mexico.

We found our answer by counting how many times the airlines were on time. We picked the one that was on time the most.

Sincerely,

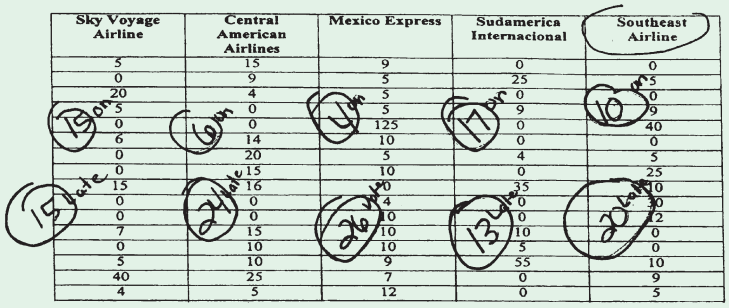

Fig. 3 Counting the number of on-time flights

If you are wanting to know the probability of the airlines being late, you must find the ratio. We did this by not counting 5 minutes and under as being late. Sky Voyage Airline's ratio, for instance, is 22/30, 22 being the times the airlines is not late and 30 being the amount of days. We found that Sky Voyage and Sudamerica Internacional are tied for first and Southeast Airlines somewhat behind at 15/30, so half the time it is late.

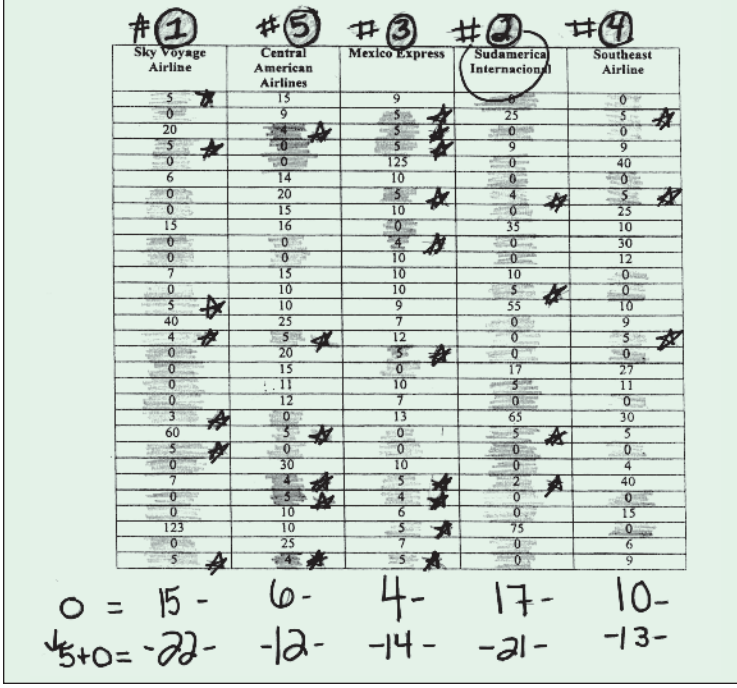

Fig. 4 Counting the number of on-time flights, with *late* being more than five minutes

Again, however, some groups refined their approach when they began to question whether a flight that leaves only five to ten minutes late is truly late. Some students recalled from experience that minimally late departures did not lead to missed connections. Thus, some groups' final revisions occurred when they decided to revise their definition of late to include only flights that left a certain number of minutes late. For example, in **figure 4**, students only considered times that were greater than five minutes late as truly late and determined the ratio of not-late departures to all departures. (Note: For Sudamerica Internacional, the students correctly highlighted the twenty-two flights that departed within five minutes or less, but they incorrectly wrote 21 under the Sudamerica Internacional column for this total.) In conclusion, the teachers learned that their students approached the case study using several different ways of thinking and by drawing on many mathematical concepts.

When teachers recognize that their students are learning, they naturally want to modify their teaching to further maximize the students' learning. As a result of this professional development experience, the teachers reported four areas of learning about teaching with worthwhile tasks such as the Kid Case Studies: tolerating student grappling, listening to students' group work to better facilitate group presentations, recognizing that learning does not end with the end of group work, and communicating task expectations to students.

The teachers learned to observe and to tolerate student grappling

Teachers learning about teaching

Most teachers and students in this project are familiar with mathematics tasks that have only one correct solution path. When students veer from the expected solution path, teachers often feel the need to assist the students by getting them back on the right path. However, when dealing with problems that have multiple solutions, such as the Kid Case Studies, too much assistance often proves counterproductive. The teachers noticed that when they provided students with hints, the students often became so focused on the hints that they left their own interpretative thought processes behind. Instead, the teachers learned to observe and to tolerate student grappling as the students engaged in the problem-solving process. In their teacher-produced handbook titled *Implementing "Case Studies for Kids" and Understanding Students' Solutions*, the teachers wrote, "As they [the students] work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution."

The teachers also realized that listening to the students' conversations gave them opportunities to learn about their students' thinking and solution processes. As a result, the teachers felt more prepared to facilitate the class discussions that accompanied the small group presentations, to ask for points of clarification, to prompt students to compare and contrast different approaches, and to facilitate students' discussions about how well the different solutions met the needs of the client. In their handbook, the teachers wrote, "Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations."

Despite the self-assessment features of Kid Case Studies, not all students progress through cycles of refinement and arrive at a more sophisticated solution. With "Departing On-Time," a source of disappointment for the teachers was that some groups only considered the average number of minutes late. However, they discovered that students could learn from one another during the follow-up presentations and discussions and that the learning did not necessarily end at the close of group work. The teachers reported that as different groups presented their solutions, the whole class began to recognize the limitations of relying on the average number of minutes. After learning of the other methods, such as using the percent of on-time departures or re-defining late, the students had the potential to interpret the problem and revise their solutions in more effective ways.

To engage the students in such productive discussions, the teachers learned to make it clear that the students were expected to carefully listen to the other groups' solutions, to try to understand the other groups' solutions, and to think about how well the different solutions met the client's needs. Some of the teachers learned to plan the order of the presentations so that multiple solution strategies emerged after two to three groups had shared their letters. The teachers then asked questions about the differences in the solutions and the effectiveness of each of the solutions in meeting the needs of the client. To further facilitate students' active listening during the group presentations, one teacher required students to explain how they would like to revise their own groups' response, having heard all of the solutions.

Conclusion

AS PART OF THE PROFESSIONAL DEvelopment experience, teachers were also asked to look back on what they had learned from these experiences and to make recommendations for how to clarify the expectations of Kid Case Studies for students. In the teacher handbook, their suggestions for other teachers to explain to students included the following:

- Case studies are longer problems; there are rarely immediate answers. You should expect to gradually revise your solution over the class period.
- There is usually more than one way to meet the needs of the client; in other words, there may be several ways to think about the problem.
- I will not answer questions such as "Is this the right way to do it?" If unsure of your solution, return to the problem and ask, "Have we thoroughly met the client's needs?"

In summary, "Departing On-Time" was a worthwhile mathematical task for students *and* became a worthwhile task for teachers because it was embedded in a professional development experience where teachers were challenged to solve student problems carefully designed to reveal students' thinking, to implement problems unfamiliar to both them and their students, and to investigate artifacts of their own practice.

References

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